Non-Interactive Zero-Knowledge from Vector Trapdoor Hash

Pedro Branco

Bocconi

Based on joint work with Arka Rai Choudhuri, Nico Döttling, Abhishek Jain, Giulio Malavolta and Akshayaram Srinivasan

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or
NIZK and Hidden-Bits Generator

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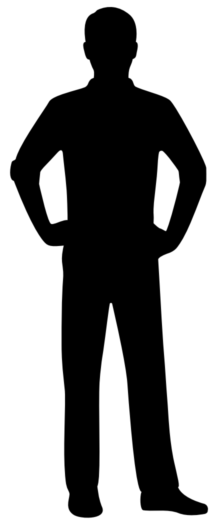
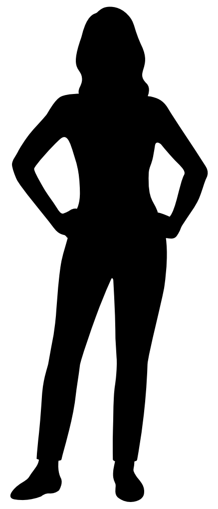
Zero-Knowledge Proofs [GMR85]



x



(x, w)

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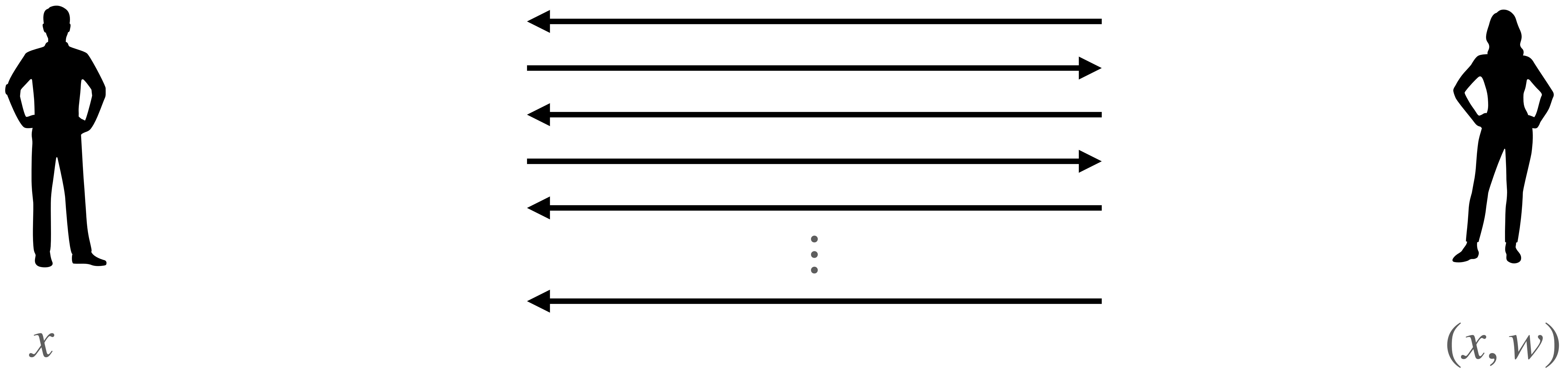

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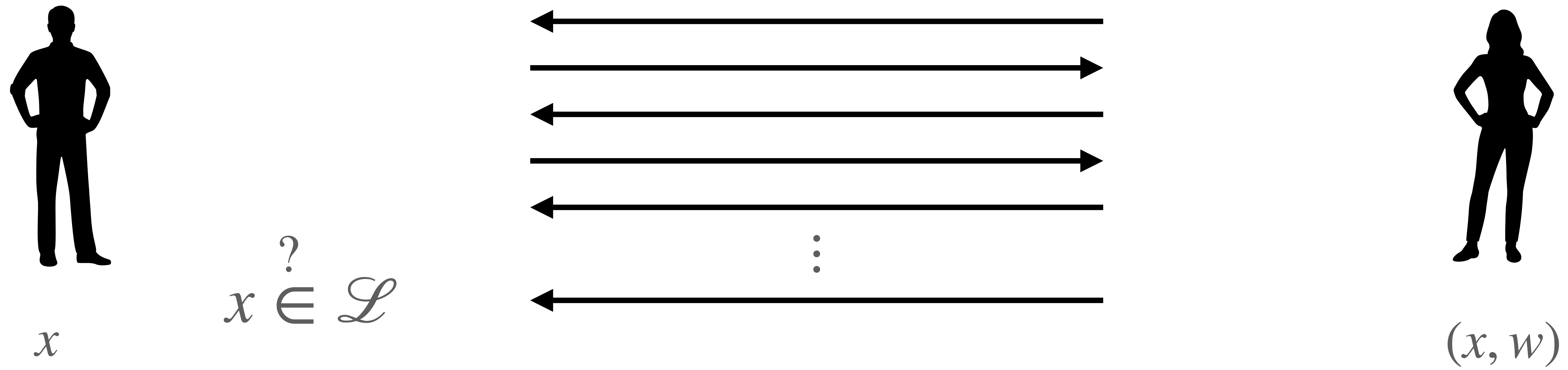

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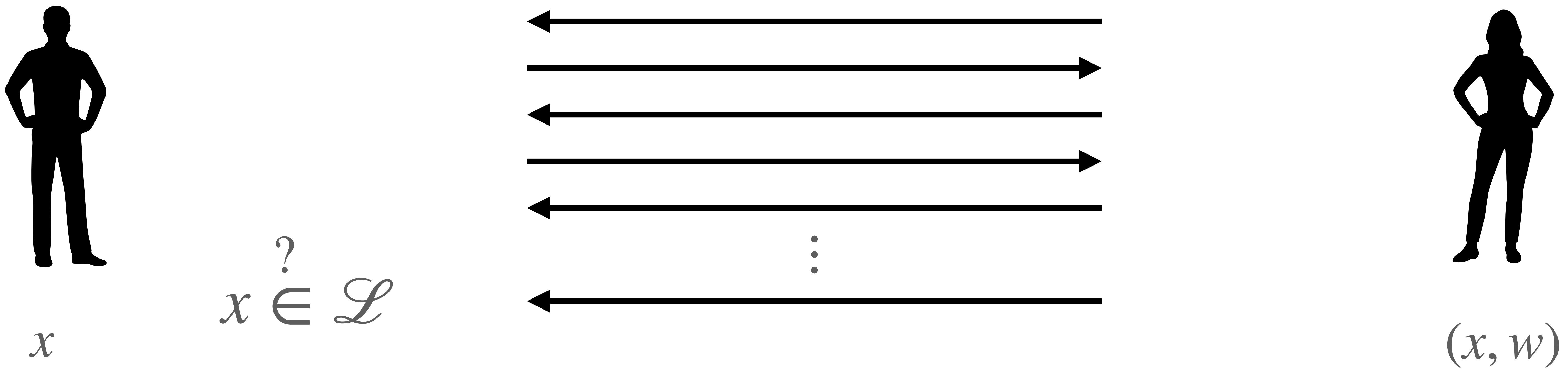

Soundness:

$$\Pr\left[x \notin \mathscr{L} \wedge \text{V accepts}\right] = \text{negl}(\lambda)$$

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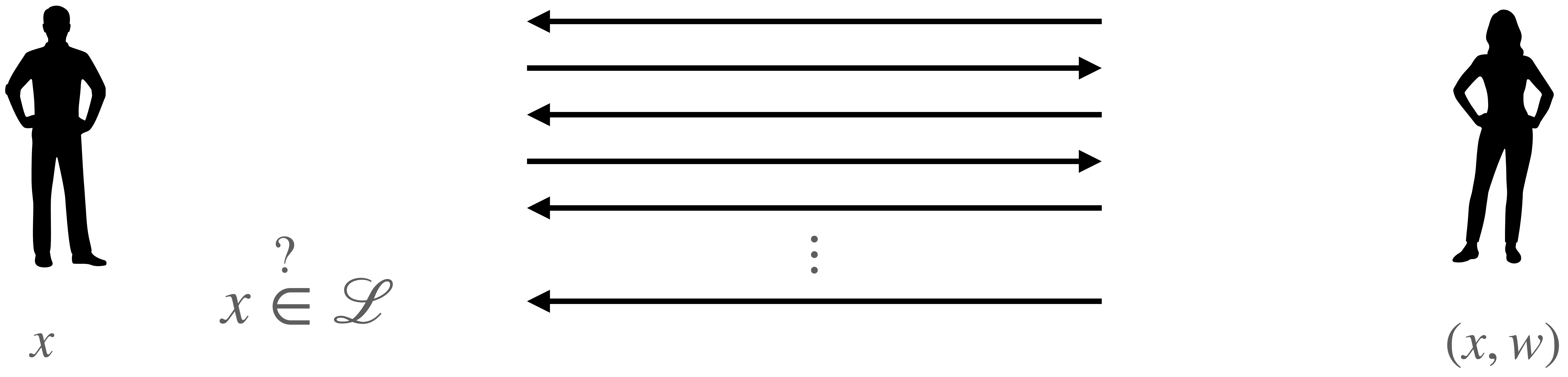

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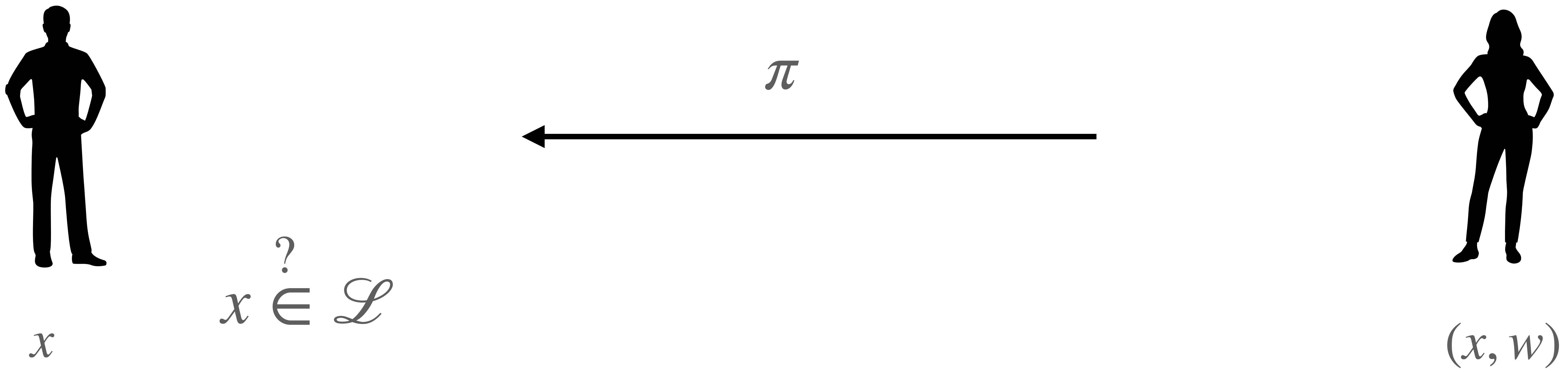
Non-Interactive Zero-Knowledge Proofs [DMP88]



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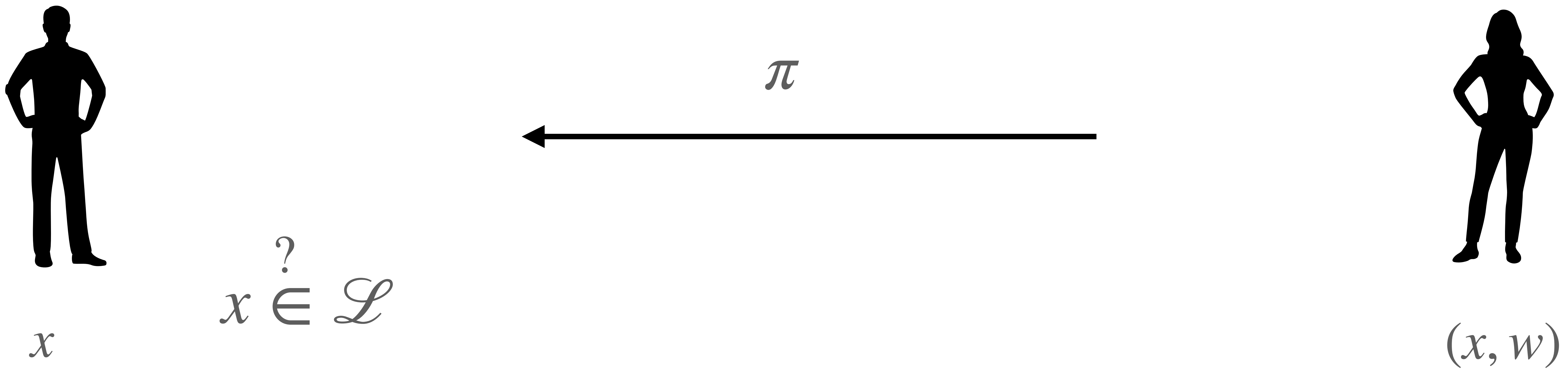

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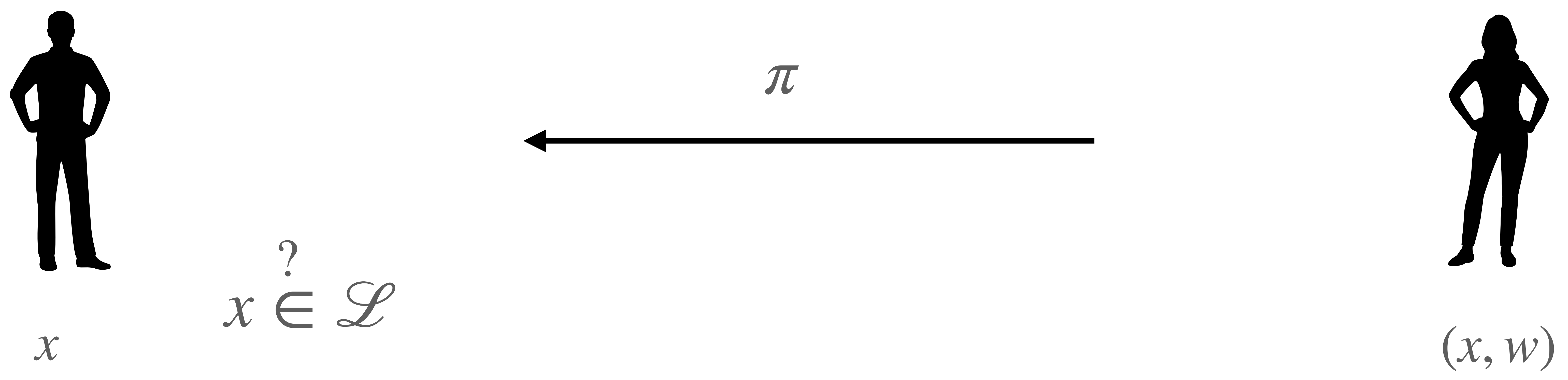

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Why NIZKs?

Theory:

- Minimal round complexity
- Minimal assumptions

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Applications:

- CCA security
- Signatures
- Blockchains

⋮

Random Oracle Vs Standard model

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Assume CRS

This talk: NIZK = NIZK for all NP in the CRS model

NIZKs Constructions

[GOS'06]

- Pairings

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Correlation Intractability Hash

- iO [CCRR18,HL18]
- FHE/LWE [CCH+19,PS19]
- DDH + LPN [BKM20]
- Sub-exp DDH [JJ21]
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Hidden-Bits Generator

- Trapdoor permutations [FLS90]
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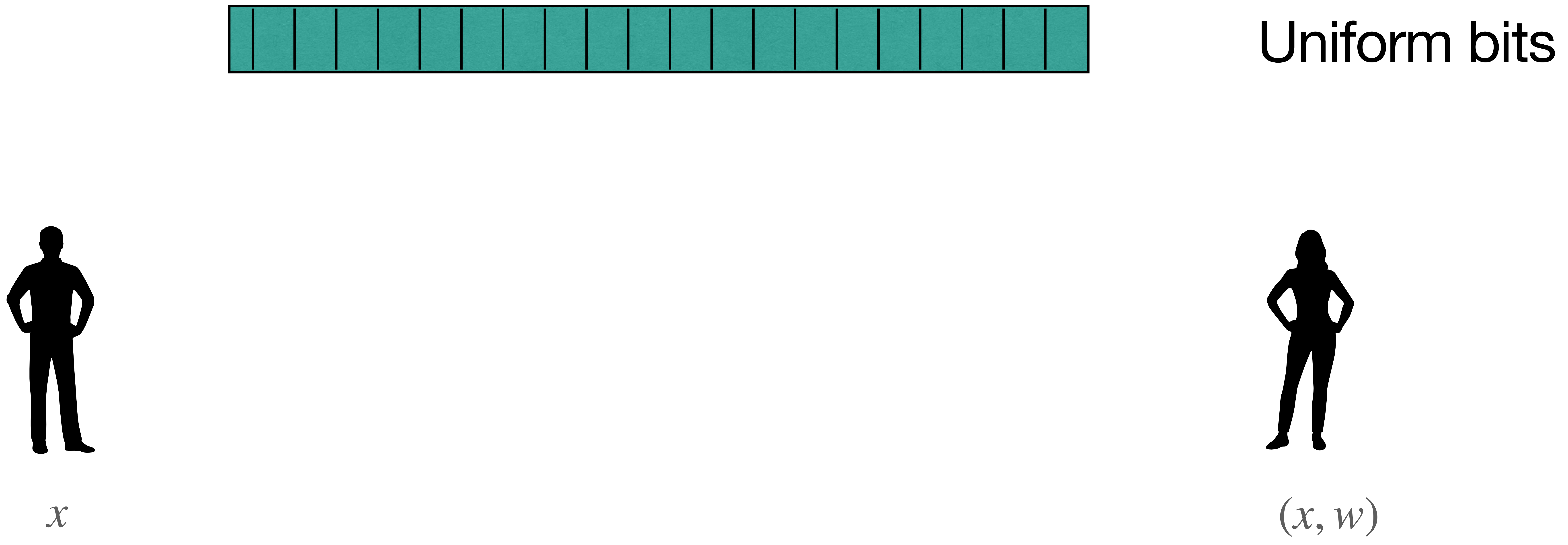

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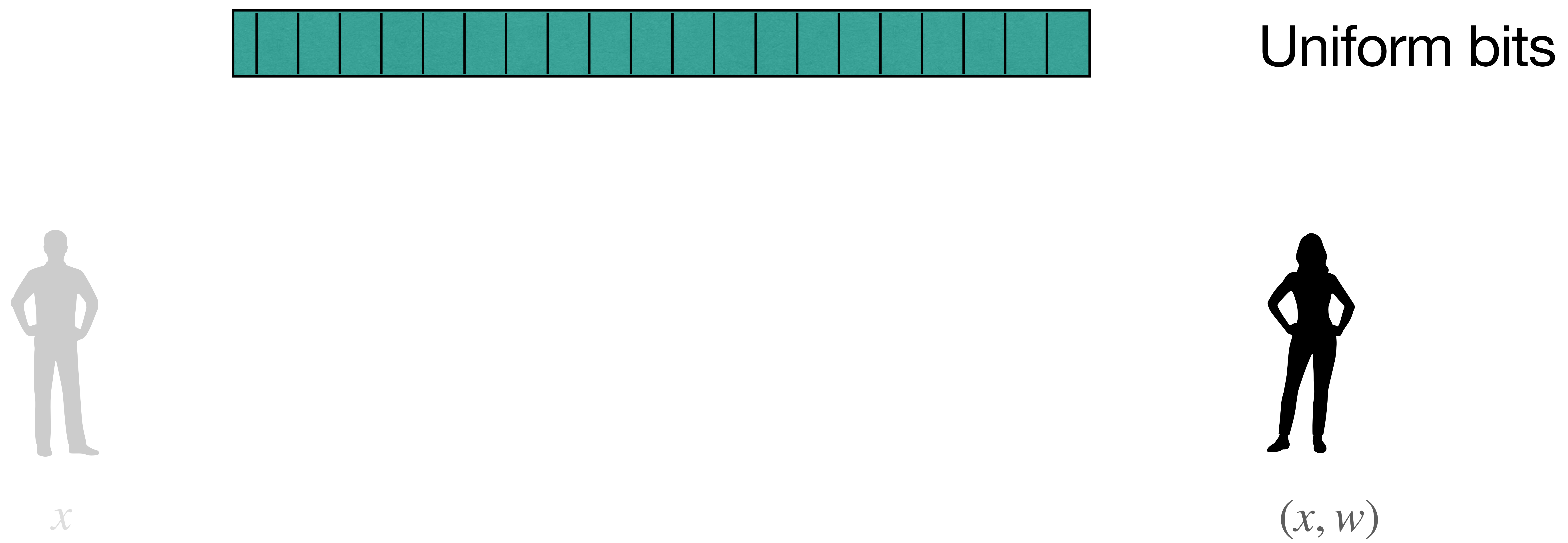

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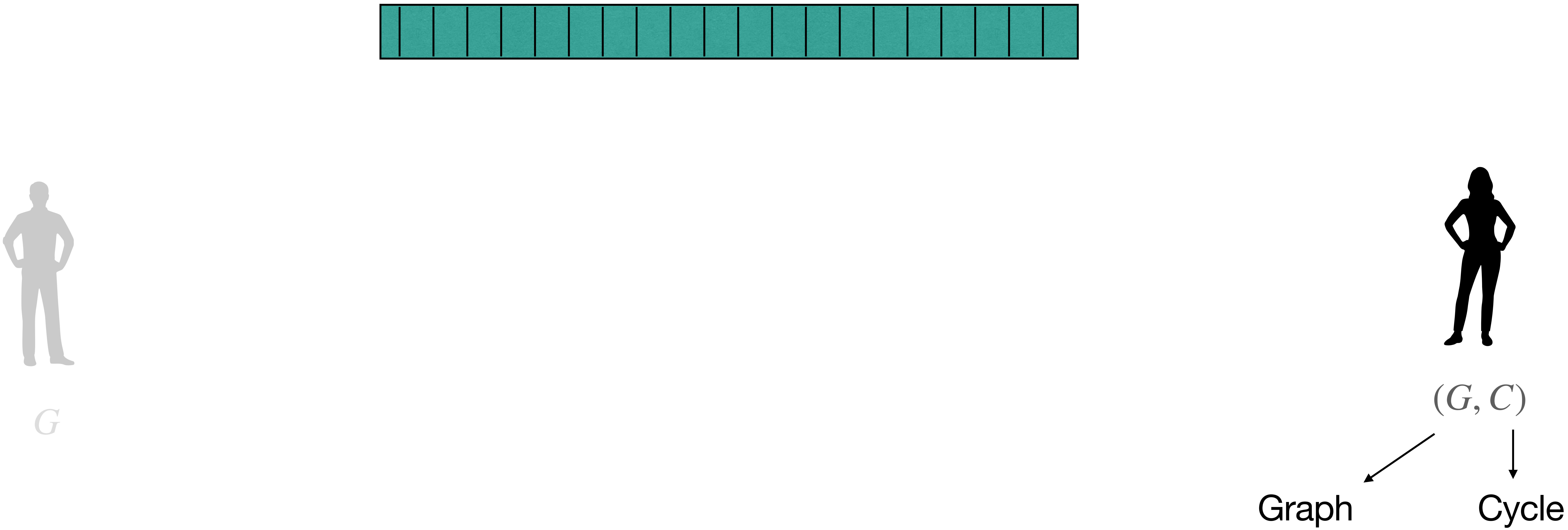
NIZK in the Hidden-Bits Model [FLS90]



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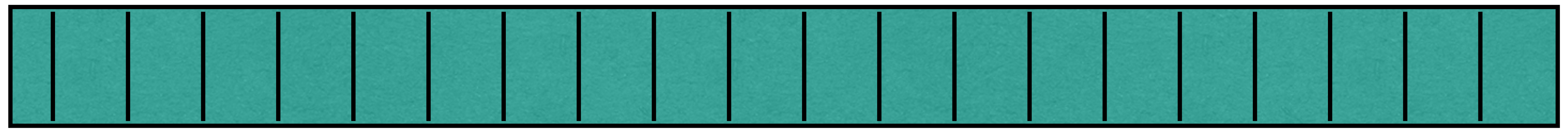
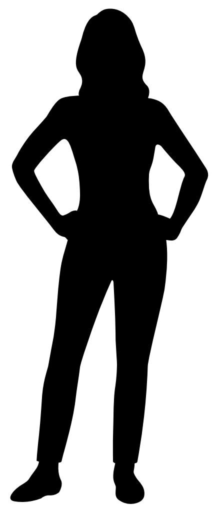


Graph H with an Hamiltonian cycle C'

G



(G, C)

Graph

Cycle

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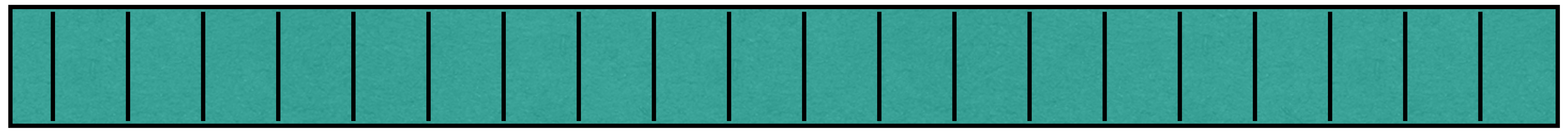

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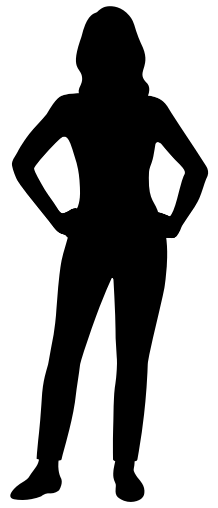
δ mapping C to C'

If e non-edge of G,
Reveal $\delta(e)$ non-edge of H



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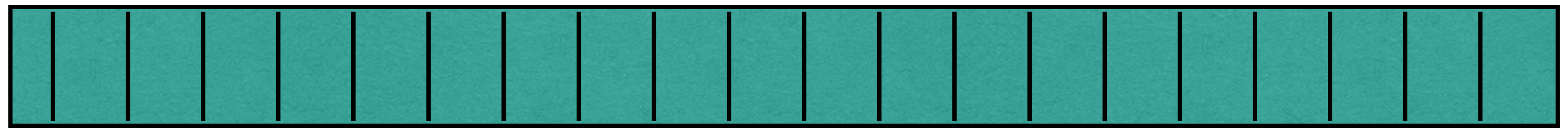

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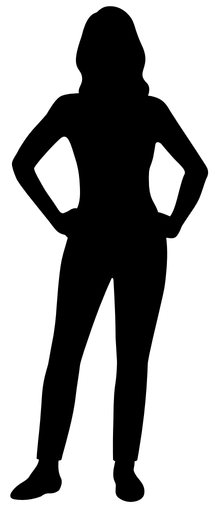

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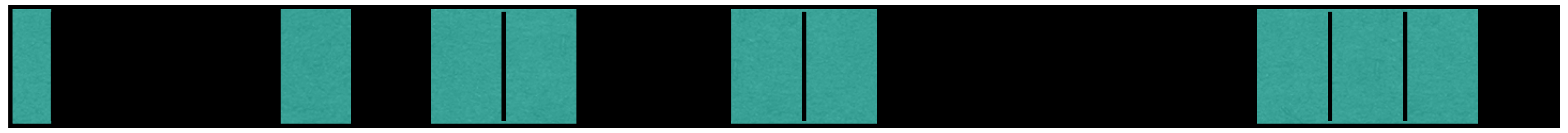

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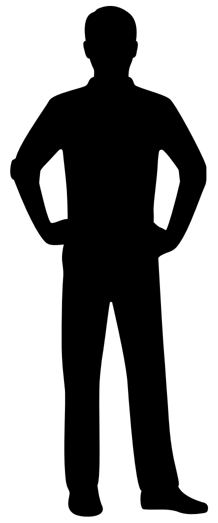

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For all non-edges e,
Check if $\delta(e)$ is non-edge

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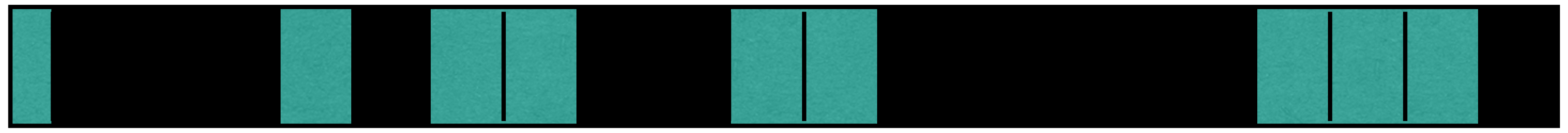

Graph H with an
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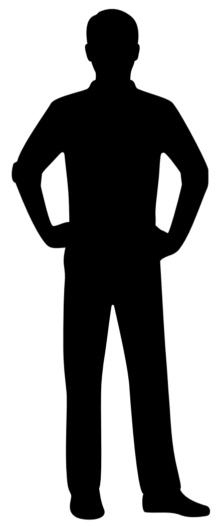
$\delta(e)$



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$$\Leftrightarrow$$

If e is an non-edge in G then $\delta(e)$ is an non-edge in H

Zero-Knowledge

Description of Sim:

- Choose a random permutation δ.

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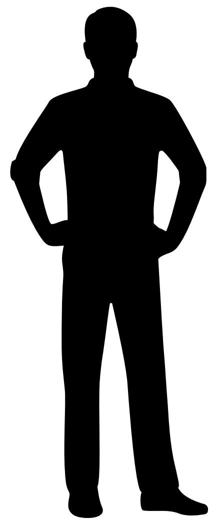
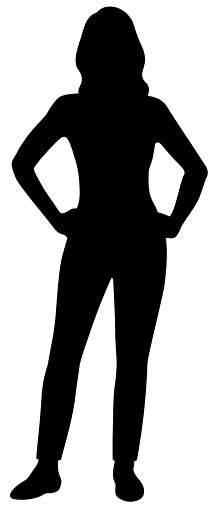
- Choose a random permutation δ.
- For all non-edges e of G, reveal a non-edge $\delta(e)$ of H
- (Sim has full control over the hidden-bits)

From Hidden-Bits Model to CRS Model:
Hidden-Bits Generator [QRW19,KMY20]

$\mathsf{crs} \leftarrow \mathsf{Setup}$





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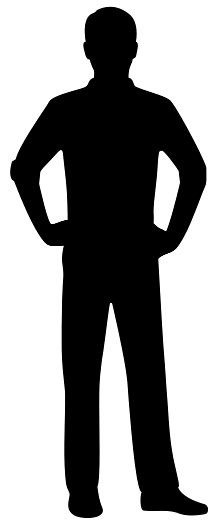
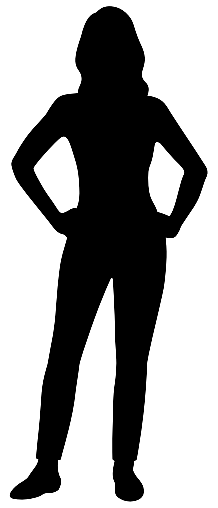


$\mathsf{com}, \{\alpha_i, \pi_i\}_{i \in S}$ ←



$(\mathsf{com}, (\alpha_1, \ldots, \alpha_k), (\pi_1, \ldots, \pi_k)) \leftarrow \mathsf{GenBits}$

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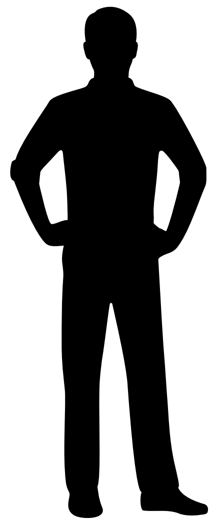

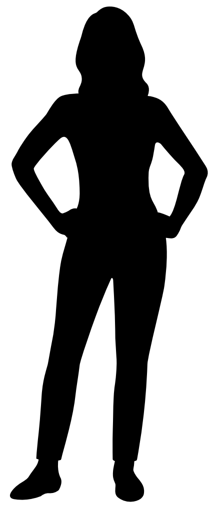

$\mathsf{Ver}(\mathsf{com}, \tilde{\alpha}, i, \pi_i)$

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Honest proofs are accepted

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Hiding:

Given com, $\{\alpha_i, \pi_i\}_{i \neq i^*}$, α_{i^*} remains hidden

Hidden-Bits Generator to NIZKs

Theorem [QRW19,KMY20]:

We can build a NIZK in the CRS model, given a HBG in the CRS model and a NIZK in the hidden-bits model

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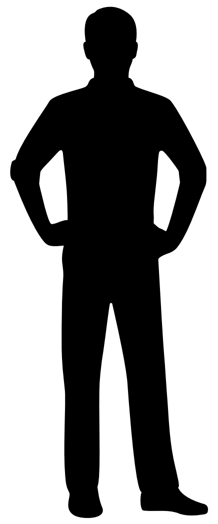
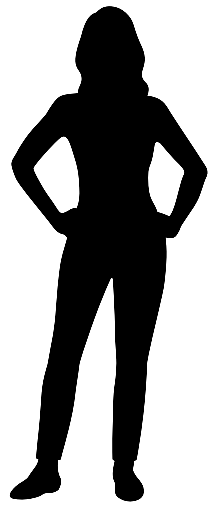
Problem:

- Build HBG.
- Previous works: TDP and LWE (with super-poly mod-to-noise).

New Primitive: Vector Trapdoor Hash





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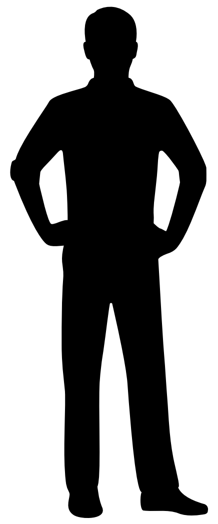
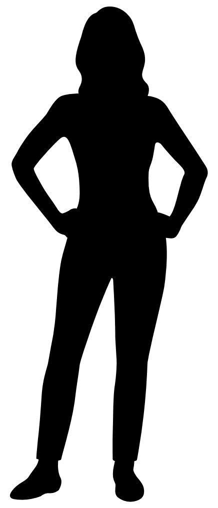
$$\left(\mathsf{hk}, \{\mathsf{ek}_i, \mathsf{td}_i\}_{i\in[k]}\right) \leftarrow \mathsf{Setup}$$





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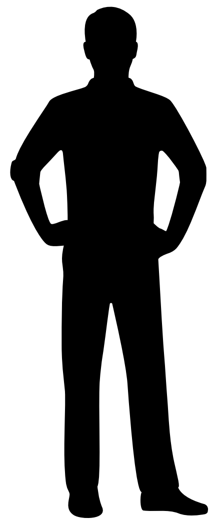

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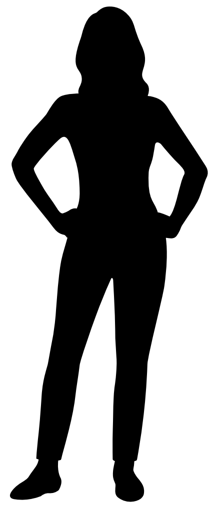

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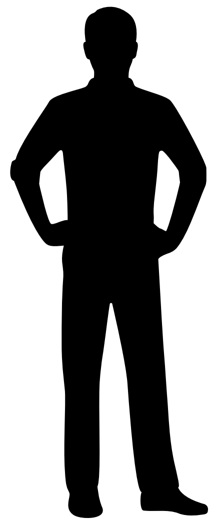

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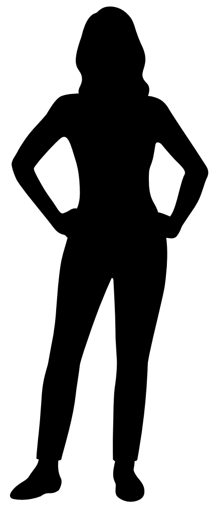

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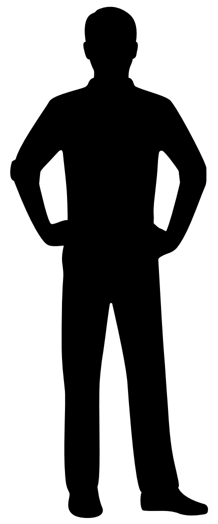

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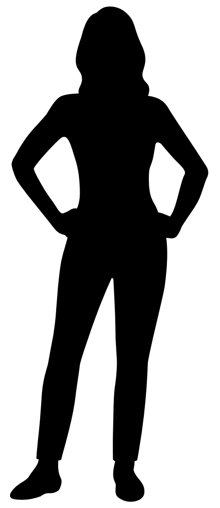

Local opening:

π_i is a local opening for $\mathbf{x}_i$

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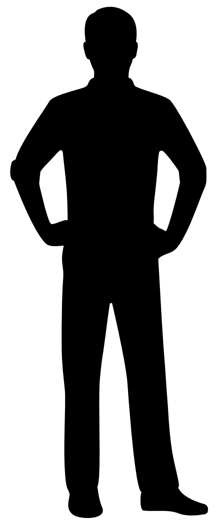

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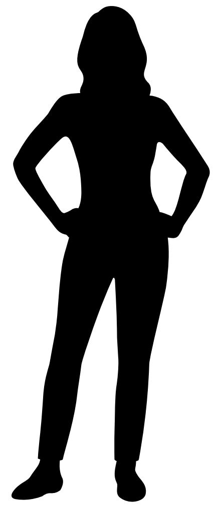

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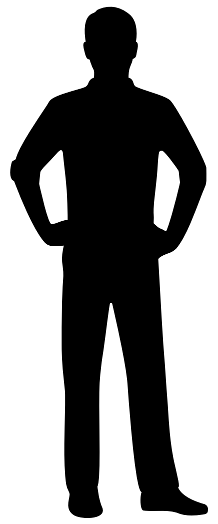

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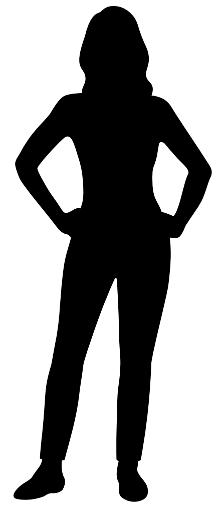

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Hiding:

e_{i^*} is uniform, given $\{e_i, \pi_i\}_{i \neq i^*}$

VTDH to Hidden-Bits Generator

Theorem:

VTDH implies HBG

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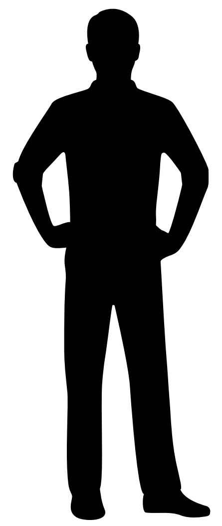
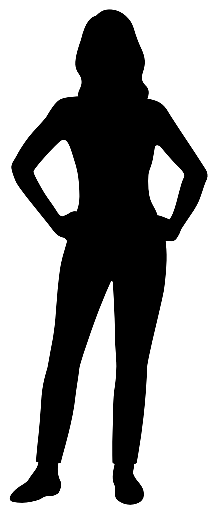
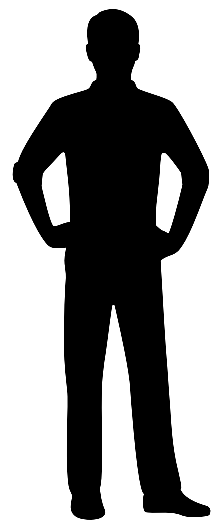




VTDH to Hidden-Bits Generator

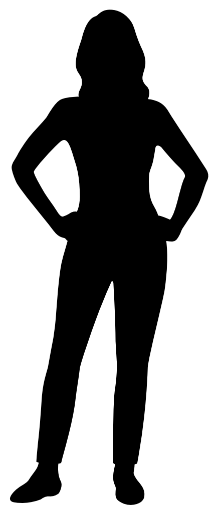
Theorem:
VTDH implies HBG

$\mathsf{hk}, \{\mathsf{ek}_i\}_{i \in [k]}$



$\left(\mathbf{h}, \{\pi_i\}_{i \in [k]}\right) \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathbf{x})$

$e_i \leftarrow \mathsf{Enc}(\mathsf{ek}_i, \pi_i)$



VTDH to Hidden-Bits Generator

Theorem:
VTDH implies HBG

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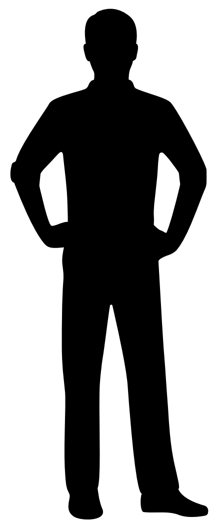

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$\mathsf{com} = \mathbf{h}$

$\alpha_i = e_i$

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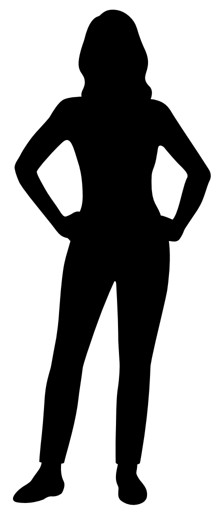

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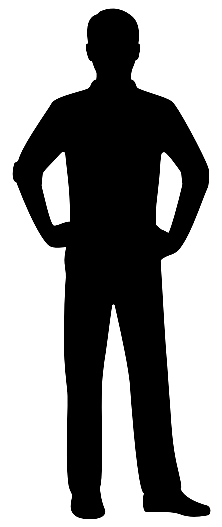

$\mathsf{com}, \{\alpha_i, \pi_i'\}_{i \in S}$ ←

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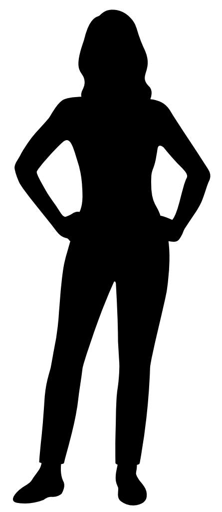

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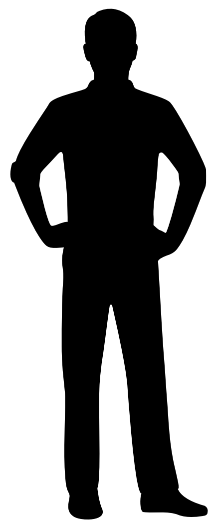

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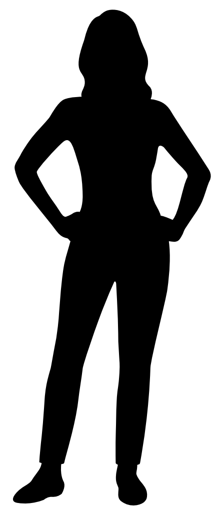

$\mathsf{LocVer}(\mathsf{com}, \pi_i, \mathbf{x}_i) = 1$

$\mathsf{Enc}(\mathsf{ek}_i, \pi_i) = e_i$

Our Results

Theorem:

VTDH from: i) LWE with polynomial mod-to-noise ratio

ii) DDH + LPN

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VTDH from: i) LWE with polynomial mod-to-noise ratio

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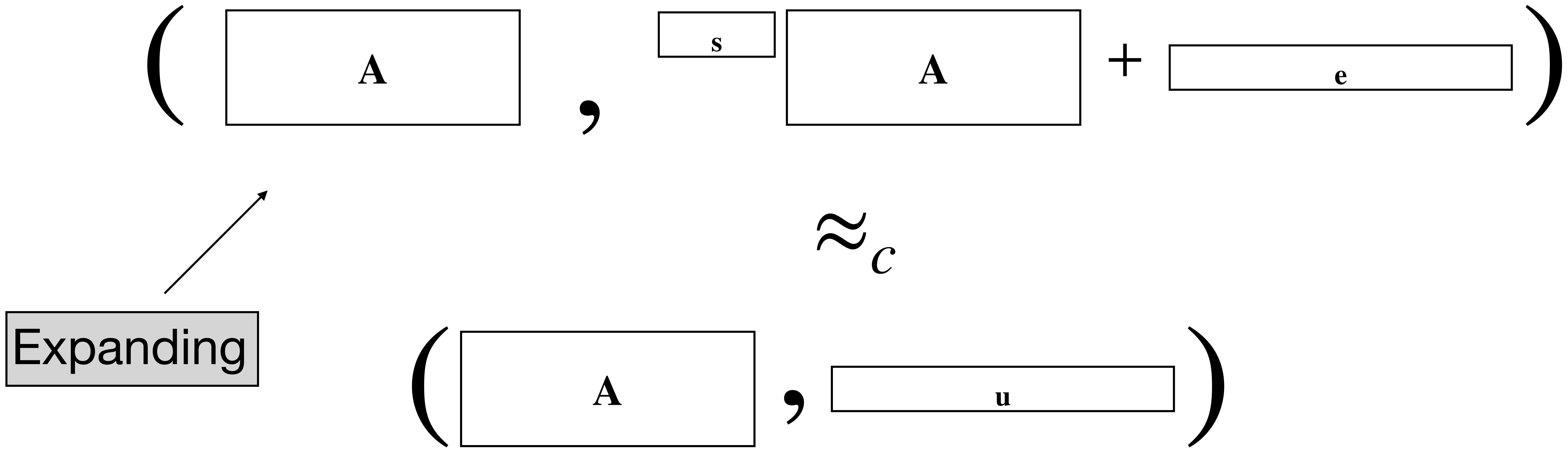
Learning with Errors



$\mathbf{A} \leftarrow \{0,1\}^{n\times m}$, $\mathbf{s} \leftarrow \{0,1\}^{n}$, $\mathbf{u} \leftarrow \{0,1\}^{m}$ and $\mathbf{e} \leftarrow \mathsf{DG}_{\sigma}^{m}$

VTDH from LWE: Hash, Encoding and Decoding

$$\mathsf{hk} = \mathbf{A}_1, \ldots, \mathbf{A}_k, \underbrace{\mathbf{W}_1, \ldots, \mathbf{W}_k}_{\text{binary}}$$

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Pick binary $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_k)$

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Local Verification:

$(\mathbf{A}_1, \ldots, \mathbf{U}_i, \ldots, , \mathbf{A}_k)\pi_i$

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$$= \sum \mathrm{U}_i \mathbf{x}_i = h$$

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Decoding: $d_i = \mathbf{s}_i^T \mathbf{h} = \mathbf{s}_i \left(\sum \mathbf{U}_i \mathbf{x}_i \right)$

$\mathsf{Round}(e_i) = \mathsf{Round}(d_i)$

$\Downarrow$

Statistical Binding

VTDH from LWE: Hiding for $i = 1$

To prove: $e_1 = \mathsf{ek}_1 \pi_1 \approx v \leftarrow \mathsf{Unif}$

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↓ LWE

$$\mathsf{ek}_i = (\mathbf{u}_1, \ldots, \mathbf{u}_i, \ldots, \mathbf{u}_k)$$

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2nd Step:

$$(\text{ek}_1 \pi_1, \mathbf{W}_1 \mathbf{x}_1) \approx_s (v, \mathbf{W}_1 \mathbf{x}_1)$$

→ LHL

Recap

- LWE Result: Dual-mode NIZK from LWE .
- DDH + LPN Result: NIZK from (DDH + LPN) with statistical soundness.

Thanks!

Non-Interactive Zero-Knowledge from Vector Trapdoor Hash

Pedro Branco

Bocconi

Based on joint work with Arka Rai Choudhuri, Nico Döttling, Abhishek Jain, Giulio Malavolta and Akshayaram Srinivasan